Quadratic Functions

Rocket Quads

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:

Students will construct a straw rocket in order to explore parabolas. Students will test different angles of launch to form a conjecture about the effect the angle of launch has on the horizontal distance traveled and the maximum height the rocket achieves. Students will then use the included Geogebra files to determine the quadratic function for each of the three launches. Using the Geogebra files, the student will create a quadratic function for the 20 degree launch, 45 degree launch, and the 80 degree launch in order to accept or reject their conjecture about the effect of the launch angle on the horizontal distance traveled and the maximum height the rocket achieves.

II. UNIT AUTHOR:

Shannon Harrell, Piedmont Governor's School of Mathematics, Science, and Technology, Henry County Public Schools/Piedmont Governor's School

III. COURSE:

Algebra II, AFDA

IV. CONTENT STRAND:

Algebra and Data Analysis

V. OBJECTIVES:

Students will test different angles of launch in order to make a conjecture about the effect of the angle of launch on the horizontal distance traveled and the maximum height. Students will use the Geogebra files of three different launch angles to test their conjecture. The learner will be able to determine the horizontal distance traveled and the maximum height of each rocket launched by using the sliders in each Geogebra file to determine the quadratic function for that launch. Once students have the quadratic function, the student can determine the maximum height by looking at the function given in vertex form, $f(x) = a(x-h)^2 + k$, to determine h and k. Students will need to determine the horizontal distance using their quadratic function by using the quadratic formula or by completing the square. Once students have the horizontal distance and maximum height for each launch, he/she can draw conclusions about their conjecture.

VI. REFERENCE/RESOURCE MATERIALS:

- Ultimate Straw Rocket Directions
- Rocket Fin template (optional)
- Ruler
- Scissors
- Hot glue gun
- Elmer's glue or similar glue
- Syringe or bike pump
    - If using a bike pump, also need a rubber tube that will fit into the nozzle of the bike pump

    - If using syringe, also need a model car fuel line
- Protractor
- Tape measurer or yard sticks
- Safety goggles
- Computers
    - Geogebra
    - Microsoft Word or similar program
    - Microsoft PowerPoint or similar program

VII. PRIMARY ASSESSMENT STRATEGIES:

Students will form a conjecture about the effect of the launch angle on the horizontal distance traveled and the maximum height using their straw rocket and different launch angles. Then, the student will use the Geogebra files of the three different launch angles to determine the quadratic function for each of the three launches. Students will then use the quadratic functions to determine the horizontal distance and maximum height of each launch. Once students have determined the horizontal distance traveled and the maximum height for each of the three launches using Geogebra, students will analyze and compare the horizontal distance to the maximum height based on each angle of launch. Students will type a summary/reflection of their findings in Microsoft Word or similar program. Also, students will create a presentation using Microsoft PowerPoint, or similar program, to convey what he/she explored and learned during this activity.

VIII. EVALUATION CRITERIA:

Students will be evaluated based on their classwork – rocket construction, staying on task, working with partner, completed activity sheet, and the three completed Geogebra files, written summary/reflection, and presentation using the attached rubric.

IX. INSTRUCTIONAL TIME:

This activity will take two or three ninety-minute class sessions.

Rocket Quads

Strand

Algebra and Data Analysis

Mathematical Objective(s)

Students will demonstrate mastery of quadratic functions by using a straw rocket and Geogebra to explore the effect of the launch angle on the horizontal distance and the maximum height. Students will create a straw rocket, launch the straw rocket using different angles, and form a conjecture about the effect of the launch angle on the horizontal distance and the maximum height. Students will then use the three Geogebra files to analyze the effect the angle has on the horizontal distance traveled and the maximum height.

Related SOL

- AII.6 (Convert between graphic and symbolic forms of functions)
- AII.7 a, b (Investigate and Analyze Functions)
- AFDA 1 b – d (Investigate and Analyze Functions)

NCTM Standards

- Recognize and apply mathematics in contexts outside of mathematics
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving
- Understand relations and functions and select, convert flexibly among, and use various representations for them
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others

Additional Objectives for Student Learning (include if relevant; may not be math-related):

Physics 1 b, c (plan and conduct investigations)
Physics 2 a, b (investigate and understand how to analyze and interpret data)

Materials/Resources

- Straws, one for every student
- Rocket Fin, one for every student (optional)
- Hot glue gun
- Elmer's glue or similar glue
- Syringe or bike pump
    - One for every two students
    - If using a bike pump, also need a rubber tube that will fit in the nozzle of the bike pump
    - If using syringe, also need a model car fuel line
- Class set of rulers
- Class set of protractors
    - one for every two students
- Class set of scissors
- Rocket Quads Activity Sheet
    - One for each student
- Rocket Quads Scoring Sheet
    - One for each student
- Rubric
    - One for each student
- Computers
    - Geogebra Files
        - located on the following website: https://sites.google.com/site/rocketquadsgeogebrafiles/
        - possible solutions can be found on the following website: https://sites.google.com/site/rocketquadspba/
    - Microsoft Word or similar program
    - Microsoft PowerPoint or similar program

Assumption of Prior Knowledge

- **Prior mathematical knowledge assumed by the task/activity.**
Students should be comfortable using Geogebra, particularly sliders, in order to determine the quadratic function that corresponds to each of the three different launch angles. Once students have found the quadratic function that corresponds to each of the three launches, they should be comfortable determining key elements of the quadratic, i.e. the vertex and zeros, in order to determine the horizontal distance traveled and the maximum height.

- **Student background knowledge.**
Students will be expected to follow the directions provided to construct their rocket and form a conjecture about the effect of the launch angle on the horizontal distance traveled and the maximum height by launching their rocket using different launch angles. Students will then use the Geogebra files in order to create the quadratic functions that correspond to each launch to verify or reject their conjecture.

- **Common ideas/themes and language**
Students will be discussing angles of launch and how it affects the horizontal distance traveled and the maximum height. Also, students should realize that the rocket travels in the path of a quadratic function; therefore, the term quadratic should emerge when students are launching the rockets.

- **Possible challenges or misconceptions.**
    - Students may find it difficult to keep the rocket in position once he/she has measured the angle for the launch if trying to launch it on their own; therefore, a partner is necessary to assist in pushing the syringe or pumping the bike pump.
    - Students may find it challenging to approximate the height of the rocket after he/she launches it, in this case the teacher may want to encourage their partner to help them approximate the height by using a landmark in the classroom as a comparison.
    - Students may find it challenging to determine the quadratic function for each of the three launches using the sliders in the Geogebra files. Student may need to manually type in values for the sliders, "a", "h", and "k". This can be done by double-clicking on the slider using the "Algebra" window on the left of the screen.

- **Relevant concepts that have already been (or should have been) explored prior to this task/activity.**
Students should be very familiar with
    - Geogebra
        - particularly using sliders
    - the general shape of a quadratic function,
    - how to determine the zeros of a quadratic function,
    - how to determine the maximum height using the vertex form of a quadratic function

- **What relevant contexts (example: analysis & impact of natural disasters; traffic control; social issues) are drawn on in relation to this concept?**
Students will gain knowledge that is relevant to the fields of engineering, physics, mathematics, and the military. The design, construction, and launch of the rocket are all important features that are explored in engineering and physics. The angle of the launch and the effect it has on the horizontal distance travelled and the maximum height are very important details explored in the military.

Introduction: Setting Up the Mathematical Task

In this activity, you will be investigating the relationship between the angle of a straw rocket when it is launched to the horizontal distance traveled and the maximum height. Instructions and guidelines are provided, but you will have to use your data to make inferences about how you believe the launch angle affects the horizontal distance and maximum height.
Consider the following questions when you are completing this activity.

- How does the angle of the rocket when launched effect the horizontal distance?
- How does the angle of the rocket when launched effect the maximum height the rocket reaches?

Once you have completed the rocket launching and Geogebra files, you will be expected to type a summary/reflection using the given criteria. Also, you and your partner will create a presentation to share your findings with the class.

Time Outline: 2 or 3 ninety minute class periods

Student Exploration

Whole Class Sharing/Discussion
Students should work with a partner to discuss everything that he/she can remember about quadratic functions and record their ideas on a piece of paper. Once all groups have finished compiling their lists, one student from each group should share one thing that they listed with the class. After every group has shared, the teacher should emphasize the common themes that emerged and discuss any unique ideas that were presented to ensure that all students have a thorough understanding of quadratic functions. This portion of the activity should take 10 – 15 minutes.

Individual Work
Students will use the "Ultimate Straw Rocket Activity" Sheet to build their own straw rocket. Each student will form their own conjecture about the effect of the launch angle on the horizontal distance traveled and the maximum height. Also, each student will write their own summary/reflection.

Small Group Work
Students will work with a partner when launching the straw rockets since it would be challenging for students to measure the angle of the rocket and keep the rocket in that position while pushing the syringe or pumping the bike pump to launch the rocket. Partners will work together so that each student will launch his/her rocket using different angles less than 90 degrees.
Once both students have launched their rocket and formed their conjecture, each pair of students will work together to complete the three Geogebra files.

Student/Teacher Actions:
Students should first work on their own to build a straw rocket. The teacher will be walking around the room to provide assistance to any student during this time; however, the teacher should encourage students that finish early to help any student near them that is having trouble. Once all rockets have been constructed, students should work with a partner to collect data from launching their rocket using different angles less than 90 degrees. Each student will form their own conjecture from launching their own rocket, however, students will work with a partner during launching to provide assistance by pushing the syringe or pumping the bike pump. The teacher should be available to answer any questions during the launching of the rockets if the students working together cannot answer their partners question(s). Students will work with their partner to complete the three Geogebra files in order to draw conclusions about their conjectures. The teacher should

be available to answer any questions while groups are working on the Geogebra files trying to determine the quadratic functions for each of the three launches.

Monitoring Student Responses
Students will share the data that was collected and explain what he/she learned from the activity in their presentation and summary/reflection. Details and ideas are provided for students for what can be included in the presentation and summary/reflection; however, it is stressed that topics listed are not the only things that students can discuss in the presentation and summary/reflection.

Assessment List and Benchmarks

Student handouts and rubric.

Ultimate Straw Rocket Activity

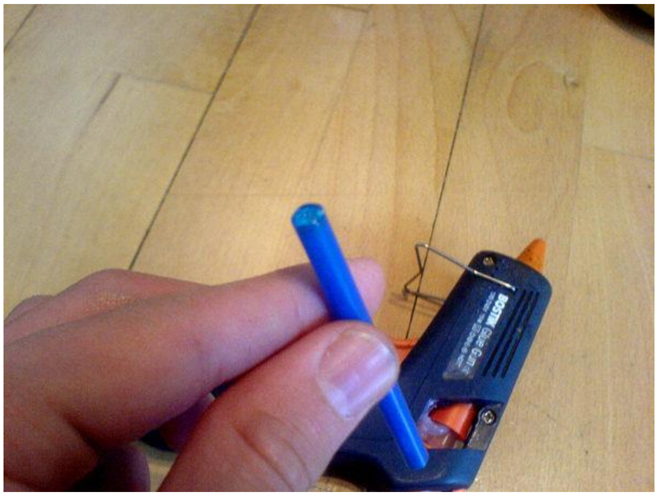
(Adapted from: http://www.instructables.com/id/Ultimate-Straw-Rocket!/ Retrieved on: May 25, 2015)

Materials

- Straws
- Rocket Fin, one for every student (optional)
- Hot glue gun
- Elmer's glue or similar glue
- Syringe or bike pump
  - One for every two students
  - If using a bike pump, also need a rubber tube that will fit into the nozzle of the bike pump
  - If using syringe, also need a model car fuel line
- Ruler
- Protractor
  - One for every two students
- Scissors
- Tape measurer or yard sticks
- Activity sheet for every student
- Rubric for every student

Making the rocket

Cut a 6cm piece of a straw.
Put a dot of hot glue in one end of the straw. Be very careful and do not touch the hot glue during this step. Make sure that it is air tight since this end will be the top of the rocket.

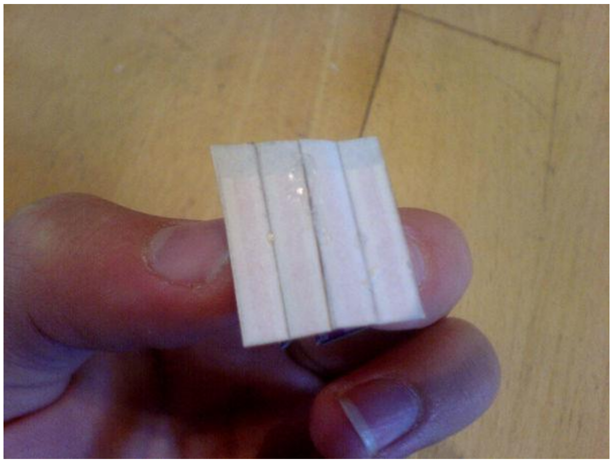
Attaching fins

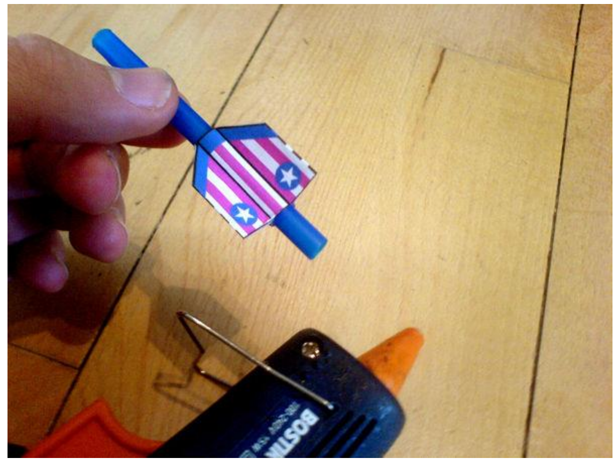
Cut out a pair of fins from the printed document.
Then fold and glue the fins together.

- Stippled lines means "fold backwards” and straight lines means the opposite.

You can use hot glue to glue the fins together, but I find that white glue (Elmer's glue or wood glue) works well.

Approximately 1-1.5 cm from the open end of the straw, the end that does not have the glue, attach the backside (the white side) of the fin and attach it around the straw using the glue gun.



Fin document provided below or it can be found at the following link: Download this .doc!

Launch it!

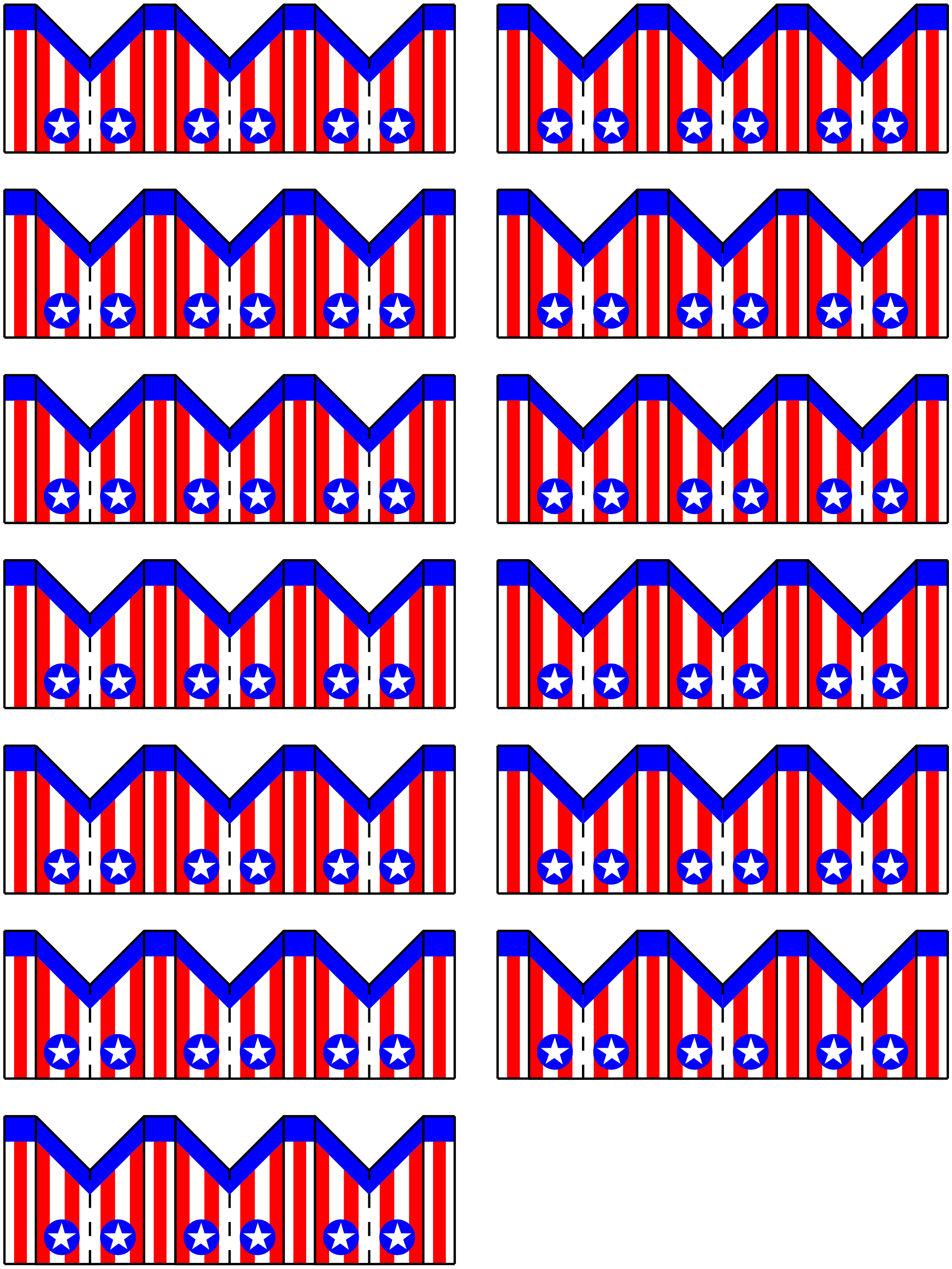
First, put on goggles.
If using a syringe to launch, connect the model car fuel line to the syringe. If using the bike pump, connect the rubber tube to the bike pump.

Place the bottom of your rocket on the fuel line on the syringe or the rubber tube that is connected to the bike pump.

Place the protractor behind the launch area.
Position your rocket at an angle less than 90 degrees using the protractor to measure the exact angle.

Give the syringe or bike pump a good, fast pump!
Now using the tape measurer or yard stick, measure the distance your rocket flew from the launching pad.
Also, estimate the height that your rocket achieved by comparing the height it reached with landmarks (on the wall) in the classroom and measure that height. This does not have to be exact, but it should help you form your conjecture.
Record this data on the Rocket Quads Activity Sheet.
Repeat this process using different angles less than 90 degrees. When you feel that you have launched the rocket enough times to create a solid conjecture, you may stop and record your hypothesis on the Rocket Quads Activity Sheet.

Fin Template Document



Ultimate Straw Rocket

- by Bunjabin

metacafe

Name:___________________________

Rocket Quads Activity Sheet

Partner's Name:___________________________

Use the following table to assist you in forming a conjecture about the effect of the angle of launch on the horizontal distance traveled and the maximum height the rocket reaches. You <u>do not</u> have to complete 8 launches, you can stop when you feel that you have launched the rocket enough times to create a solid conjecture. Record your hypothesis on this activity sheet.

Launch	Launch Angle	Horizontal Distance Traveled	Estimate of Height Achieved	Other
1				
2				
3				
4				
5				
6				
7				
8				

Now put it all together….

Use your data to answer the following questions in your own words:

- How does the angle of the rocket when launched effect the horizontal distance?

- How does the angle of the rocket when launched effect the maximum height the rocket reaches?

Time to test your theory....

- Work with your partner to complete the three Geogebra files found on the following website. https://sites.google.com/site/rocketquadsgeogebrafiles/
- Complete each of the three files for the different launch angles for the straw rocket. Once you have completed each file, save it as "LastName_LastName_FileName" using each person's last name and the original file name.
  - Example: "Harrell_Royall_20 Degree Launch"
- You and your partner should work together to determine the distance and maximum height that the rocket attained in the "20 Degree Launch" Geogebra file. Be sure to record your data and all calculations on this sheet, on the Geogebra file using a textbox, or on a separate sheet of paper.
  - Repeat this process for the "45 Degree Launch" and the "80 Degree Launch" Geogebra files.

- Using what you discovered in the Geogebra files, draw conclusions about your conjecture. Do you agree or disagree with your original hypothesis? Why? (Explain your reasoning.)

Time to reflect and share....

- Each pair of students will present their findings to the class. The presentation should include, but is not limited to, what you discovered during this activity, a discussion of how your group determined the horizontal distance and the maximum height for each of the three launches using the Geogebra files, any challenges that you encountered, and how this activity relates to the real-world besides launching rockets. Presentations should not exceed six minutes.

- Each student should turn in a typed reflection summarizing your findings, all calculations, and what those calculations represent. Please understand that the summary is separate from the presentation, and your thoughts should be conveyed in your summary as well as in your presentation. In other words, they are each "stand alone pieces" of the project. Your summary should include what you discovered during this activity, a discussion of how you determined the horizontal distance and the maximum height for each of the three launches using the Geogebra files, if you agreed or disagreed with your original conjecture and why, a discussion of any challenges you encountered, how this activity relates to the real-world besides launching rockets, and a summary of what you learned during this project. The document should be typed in Microsoft Word, or similar program, using 12 point font, double spaced and should be a **minimum** of 300 words.

Name:___________________________

Partner's Name: _______________________

Rocket Quads Scoring Sheet

Number	Element	Point Value	Self-Graded	Teacher Grade		Total
1	Preliminary Classwork	4			points earned x 2.5	
2	Work ethic and attitude while working with partner.	4			points earned x 2.5	
3	Completed Activity Sheet	4			points earned x 2.5	
4	Presentation with Partner	4			points earned x 2.5	
5	Completed Geogebra Files	4			points earned x 5	
6	Analysis of Conjecture and Findings	4			points earned x 5	
7	Summary/Reflection	4			points earned x 5	
					Final Grade	

Elements 1 – 4 are worth 10 points each.

Elements 5 – 7 are worth 20 points each.

Your grade for each element will be determined using the following rubric.

Rocket Quads Scoring Sheet

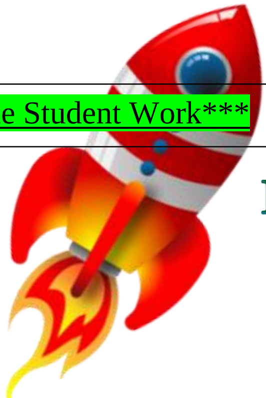
Element	Excellent (4 points)	Good (3 points)	Fair (2 points)	Poor, but attempted (1 point)	Did not complete (0 points)
1. Preliminary Classwork **- Construction of Straw Rocket** **- Staying on Task**	Followed all directions during construction of straw rocket and stayed on task.	Followed all directions during construction of straw rocket; however, student needed help staying on task on one occasion.	Followed all directions during construction straw rocket; however, student needed help staying on task on several occasions (2 – 4 times).	Did not follow all directions during construction and student needed help staying on task on several occasions (2 – 4 times).	Did not construct a straw rocket and did not stay on task.
2. Work ethic and attitude while working with partner.	Assisted partner during rocket launching and presentation by contributing and cooperating the entire time. Teacher did not have to intervene in order for you to contribute and cooperate.	Assisted partner during rocket launching and presentation by contributing and cooperating most of the time. Teacher had to intervene on 1 occasion in order for you to contribute and cooperate.	Assisted partner during rocket launching and presentation by contributing and cooperating some of the time. Teacher had to intervene on 2 occasions in order for you to contribute and cooperate.	Did not work well with partner and did not assist during rocket launching. Teacher had to intervene on 3 occasions in order for you to contribute and cooperate.	Did not contribute or cooperate with partner in order to complete launching and the presentation.
3. Activity Sheet Completion	Every portion of the activity sheet is completed. All work and calculations are accurate.	Every portion of the activity sheet is completed. Most work and calculations are accurate (1 – 2 errors).	Most of the activity sheet is completed, one section incomplete. All work and calculations are accurate.	Most of the activity sheet is completed, one section incomplete. Most work and calculations are accurate (1 – 2 errors).	Did not attempt to complete the activity sheet.
4. Presentation with Partner (4 – 6 minutes)	• Relaxed and self-confident • Builds trust and holds attention by direct eye contact with audience • Fluctuation in volume and inflection help to maintain interest and emphasize key points.	• Quick recovery from minor mistakes • Fairly consistent use of direct eye contact with audience • Satisfactory variation of volume and inflection. • Design and visual aids effectively used	• Some tension or indifference apparent • Occasional but inconsistent eye contact with audience • Uneven volume with little or no inflection. • Design and visual aids do not enhance presentation.	• Nervous tension obvious and/or lack of interest in topic • No effort to make eye contact with audience • Low volume and/or monotonous tone cause audience to disengage. • Design and visual aids were not present in presentation.	Did not attempt the presentation.

	• Design and visual aids effectively used to enhance presentation.	to enhance presentation.			

Element	Excellent (4 points)	Good (3 points)	Fair (2 points)	Poor, but attempted (1 point)	Did not complete (0 points)
5. Completion of Geogebra Files	Accurately determined a Quadratic function for all 3 Geogebra files. Calculations for horizontal distance traveled and maximum height present with no errors.	Accurately determined a Quadratic function for all 3 Geogebra files. Calculations for horizontal distance traveled and maximum height present with 1 – 2 errors.	Accurately determined a Quadratic function for all 3 Geogebra files. Calculations for horizontal distance traveled and maximum height present with 3 – 4 errors.	Quadratic function for all 3 Geogebra files found, but 1 does not accurately represent the data, or calculations for horizontal distance traveled and maximum height present with more than 4 errors.	Did not attempt to complete the Geogebra files.
6. Analysis of Conjecture and Findings	Draws clear and valid conclusions and inferences are supported by data and calculations in presentation and summary.	Draws valid conclusions and inferences are somewhat supported by data and calculations in presentation and summary.	Draws valid conclusions but inferences are not supported by data and calculations in presentation or summary.	Draws inaccurate conclusions and inferences in presentation or summary. Data and calculations not present in presentation or summary.	Did not analyze and form conclusions about conjecture in the presentation or summary.
7. Summary/ Reflection	Exhibits strong knowledge of topics explored using in-depth explanations and calculations. Free of grammatical errors.	Exhibits strong knowledge of topics explored using in-depth explanations and calculations, but contains 1 – 3 grammatical errors.	Exhibits some knowledge of topics explored by providing essential calculations, but explanations could be improved. Contains 1– 3 grammatical errors.	Exhibits little knowledge of topics explored since explanations were lacking and calculations were not present, or contains more than 5 grammatical errors.	Did not write a summary/ reflection.

Feedback:

Adapted from: http://www.slideshare.net/ArmandoCastillo2/oral-presentation-rubric2



Sample Student Work

Name: ***Sample Student Work***

Rocket Quads Activity Sheet

Partner's Name: ***Sample Student Work***

Use the following table to assist you in forming a conjecture about the effect of the angle of launch on the horizontal distance traveled and the maximum height the rocket reaches. You do not have to complete 8 launches, you can stop when you feel that you have launched the rocket enough times to create a solid conjecture. Record your hypothesis on this activity sheet.

Launch	Launch Angle	Horizontal Distance Traveled	Estimate of Height Achieved	Other
1	30 degrees	86 inches	40 inches	
2	50 degrees	77 inches	44 inches	The rocket bounced when it landed (but it did not move more than about an inch)
3	70 degrees	65 inches	53 inches	
4				
5				
6				
7				
8				

I used 20 cc of air for each launch so that it would be consistent. I stopped after 3 launches because I believe I can tell how the angle of launch is affecting the horizontal distance and the maximum height.

Now put it all together….

Use your data to answer the following questions in your own words:

- How does the angle of the rocket when launched effect the horizontal distance?
  I think that the smaller the angle, the farther the rocket will travel. As the angle increased, the horizontal distance the rocket traveled decreased.

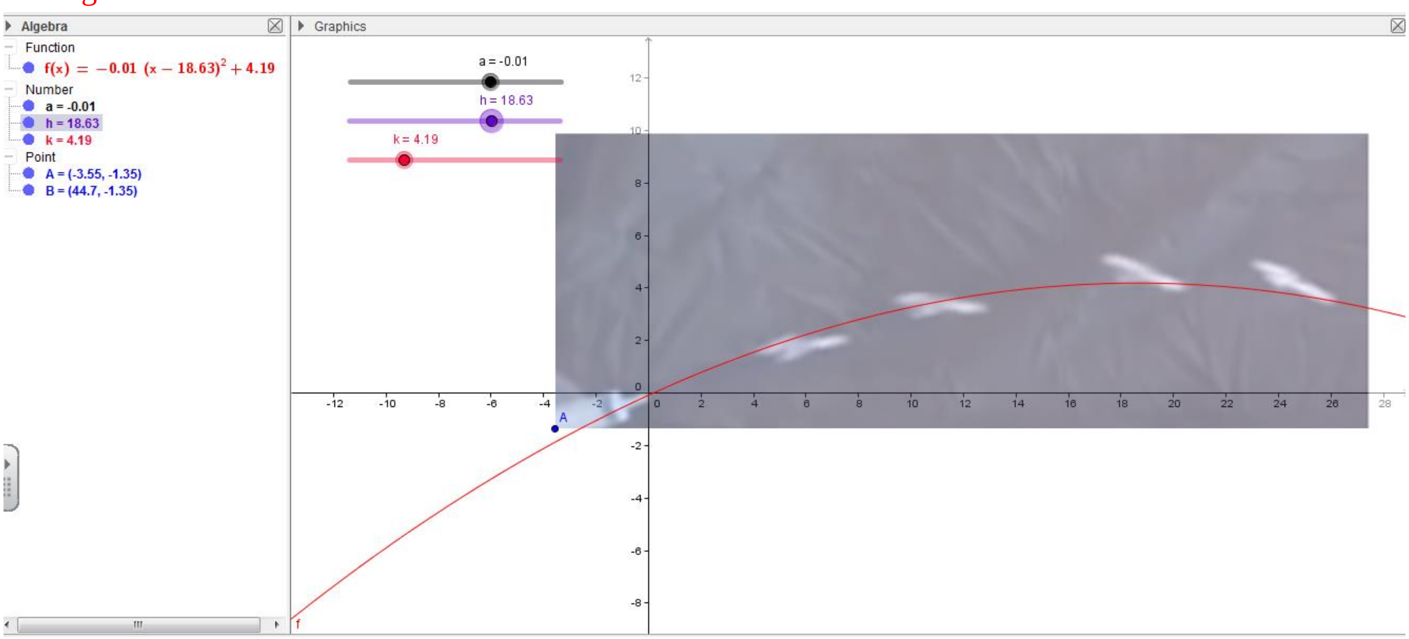
- How does the angle of the rocket when launched effect the maximum height the rocket reaches?
  The larger the angle, the higher the rocket flew. As the angle increased, the maximum height of the rocket increased. I could see this even though I could not get the exact measurement, I approximated how high the rocket reached by comparing it to the book shelf since I was launching my rocket in front of the bookshelf.

Time to test your theory….

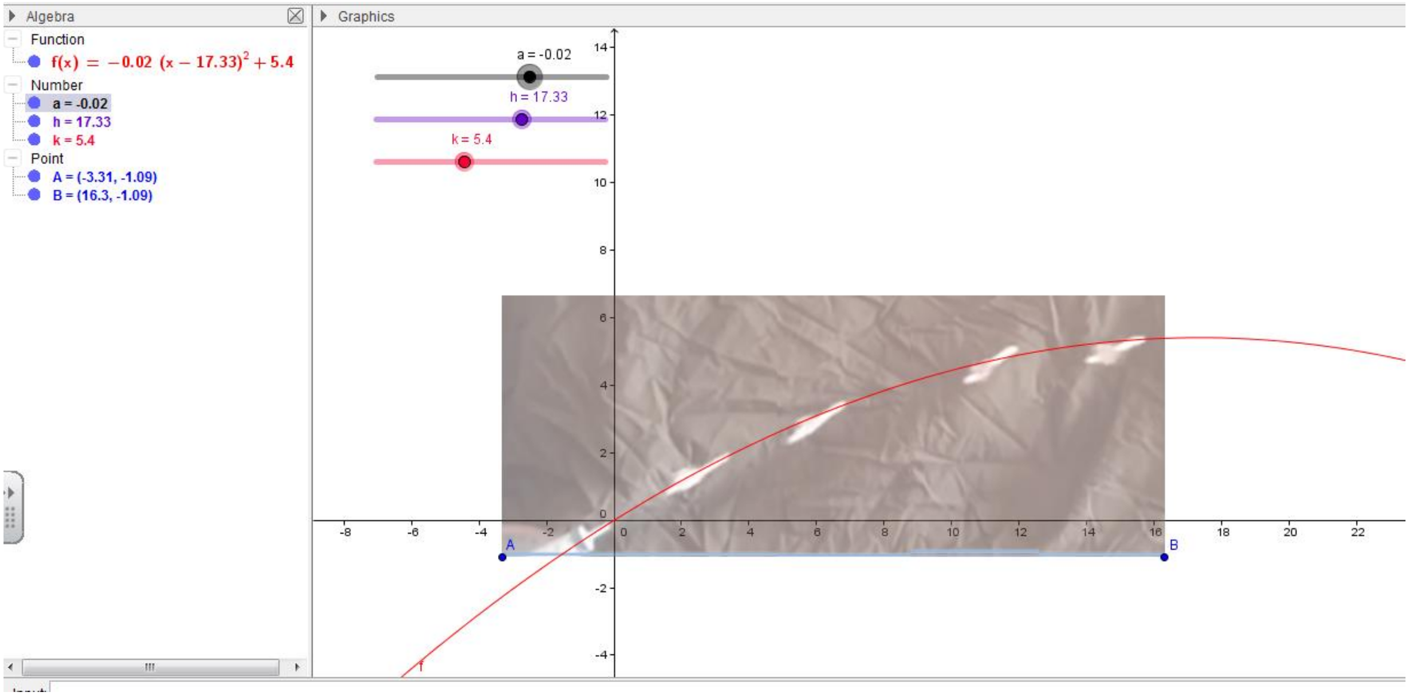
- Work with your partner to complete the three Geogebra files found on the following website. https://sites.google.com/site/rocketquadsgeogebrafiles/
- Complete each of the three files for the different launch angles for the straw rocket. Once you have completed each file, save it as "LastName_LastName_FileName" using each person's last name and the original file name.
  - Example: "Harrell_Royall_20 Degree Launch"

These possible solutions can be found at the following website:
https://sites.google.com/site/rocketquadspba/home/geogebra-files

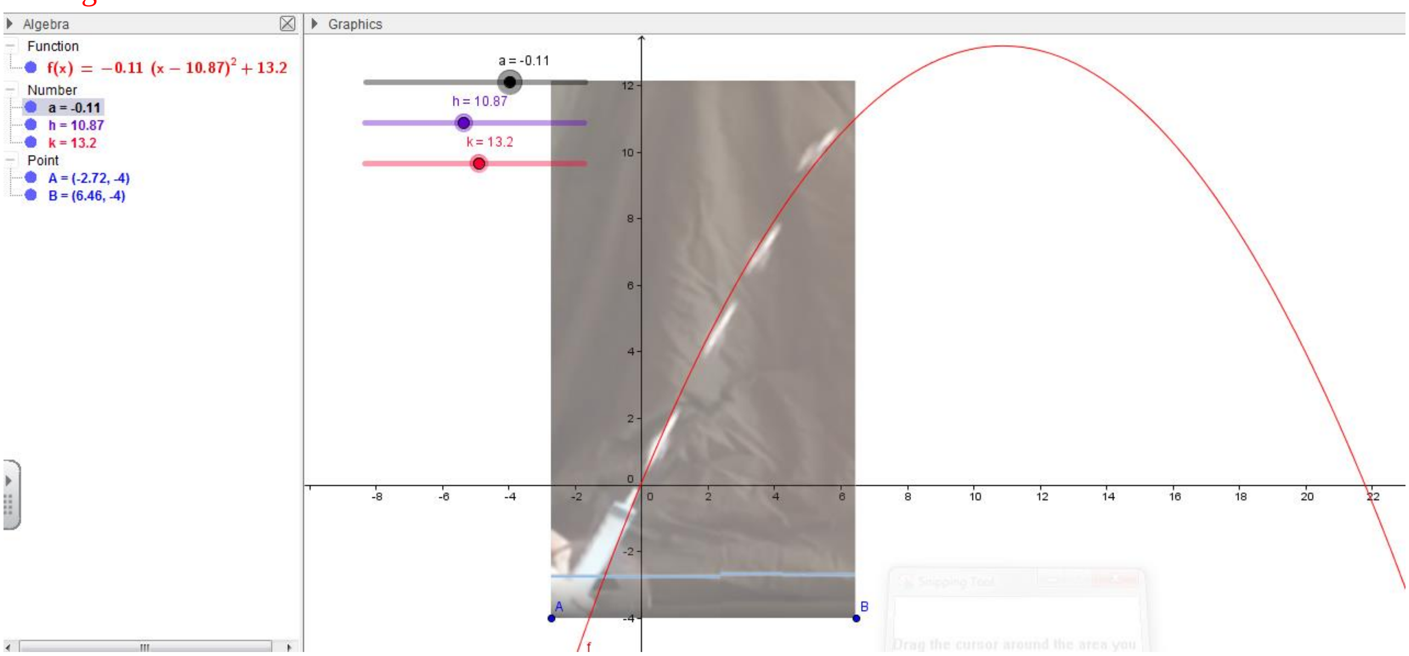
Screenshots of the files:
20 Degree Launch Possible Solution



45 Degree Launch Possible Solution



80 Degree Launch Possible Solution



- You and your partner should work together to determine the distance and maximum height that the rocket attained in the "20 Degree Launch" Geogebra file. Be sure to record your data and all calculations on this sheet, on the Geogebra file using a textbox, or on a separate sheet of paper.
  - Repeat this process for the "45 Degree Launch" and the "80 Degree Launch" Geogebra files.

20 Degree Launch

Exact "a" value found by right-clicking on the "a" slider in the window on the left and selecting "Object Properties" and then click the tab "Basic". This was done since the Algebra window was listing "a= -0.01".

$f(x) = -0.0123(x - 18.63)^2 + 4.19$

$f(x) = -0.0123(x^2 - 37.26x + 347.1769) + 4.19$

$f(x) = -0.0123x^2 + 0.458298x - 4.26904587 + 4.19$

$f(x) = -0.0123x^2 + 0.458298x - 0.07904587$

Now use the quadratic formula to find the largest zero, which will be the horizontal distance traveled.

$$x = \frac{-0.458298 \pm \sqrt{(0.458298)^2 - 4(-0.0123)(-0.07904587)}}{2(-0.0123)}$$

$$x = \frac{-0.458298 \pm \sqrt{0.2100370568 - 0.0038890568}}{-0.0246}$$

$$x = \frac{0.458298 \pm \sqrt{0.206148}}{0.0246}$$

$x = \frac{0.458298 - \sqrt{0.206148}}{0.0246}$ and $x = \frac{0.458298 + \sqrt{0.206148}}{0.0246}$

$x = 0.1732828902$ units and $x = 37.08671711$ units

Therefore, the horizontal distance traveled is $x = 37.08671711$ units. This is verified by looking at the Geogebra file, since the parabola crosses between 35 and 40.

$f(x) = -0.0123(x - 18.63)^2 + 4.19$, so the vertex is (18.63, 4.19).

Therefore, the maximum height is 4.19 units.

45 Degree Launch

Exact "a" value found by right-clicking on the "a" slider in the window on the left and selecting "Object Properties" and then click the tab "Basic". This was done since the Algebra window was listing "a= -0.02".

$f(x) = -0.018(x - 17.33)^2 + 5.4$

$f(x) = -0.018(x^2 - 334.66x + 300.3289) + 5.4$

$f(x) = -0.018x^2 + 0.62388x - 5.4059202 + 5.4$

$f(x) = -0.018x^2 + 0.62388x - 0.0059202$

Now use the quadratic formula to find the largest zero, which will be the horizontal distance traveled.

$$x = \frac{-0.62388 \pm \sqrt{(0.62388)^2 - 4(-0.018)(-0.0059202)}}{2(-0.018)}$$

$$x = \frac{-0.62388 \pm \sqrt{0.3892262544 - 0.0004262544}}{-0.036}$$

$$x = \frac{0.62388 \pm \sqrt{0.3888}}{0.036}$$

$x = \frac{0.62388 - \sqrt{0.3888}}{0.036}$ and $x = \frac{0.62388 + \sqrt{0.3888}}{0.036}$

$x = 0.009491925$ units and $x = 34.65050808$ units

Therefore, the horizontal distance traveled is $x = 34.65050808$ units. This is verified by looking at the Geogebra file, since the parabola crosses between 34 and 36.

$f(x) = -0.018(x - 17.33)^2 + 5.4$, so the vertex is (17.33, 5.4).
Therefore, the maximum height is 5.4 units.

80 Degree Launch
Exact "a" value found by right-clicking on the "a" slider in the window on the left and selecting "Object Properties" and then click the tab "Basic". This was done since the Algebra window was listing "a= -0.11".

$f(x) = -0.111(x - 10.87)^2 + 13.2$

$f(x) = -0.111(x^2 - 21.74x + 118.1569) + 13.2$

$f(x) = -0.111x^2 + 2.41314x - 13.1154159 + 13.2$

$f(x) = -0.111x^2 + 2.41314x + 0.0845841$

Now use the quadratic formula to find the largest zero, which will be the horizontal distance traveled.

$$x = \frac{-2.41314 \pm \sqrt{(02.41314)^2 - 4(-0.111)(0.0845841)}}{2(-0.111)}$$

$$x = \frac{-2.41314 \pm \sqrt{5.82324466 + 0.0375553404}}{-0.222}$$

$$x = \frac{2.41314 \pm \sqrt{5.8608}}{0.222}$$

$x = \frac{2.41314-\sqrt{5.8608}}{0.222}$ and $x = \frac{2.41314+\sqrt{5.8608}}{0.222}$

$x = -0.0349951351$ units and $x = 21.77499514$ units

Therefore, the horizontal distance traveled is $x = 21.77499514$ units. This is verified by looking at the Geogebra file, since the parabola crosses between 20 and 22.

$f(x) = -0.111(x - 10.87)^2 + 13.2$, so the vertex is (10.87, 13.2).
Therefore, the maximum height is 13.2 units.

- Using what you discovered in the Geogebra files, draw conclusions about your conjecture. Do you agree or disagree with your original hypothesis? Why? (Explain your reasoning.)

The Geogebra files helped me to confirm my conjecture since the 20 degree launch had the longest distance of 37.08671711 units, but the shortest maximum height of 4.19 units. The 80 degree launch had the shortest distance of 21.77499514 units, but it reached the highest with a height of 13.2 units. The 45 degree launch had a horizontal distance that fell between the distances of the 20 degree launch and the 80 degree launch since it was 34.65050808 units. Also, the 45 degree launch reached a height between the height of the 20 degree launch and the 80 degree launch heights by reaching 5.4 units. The larger angles will reach higher heights, but they will not travel as far horizontally. The smaller angles will travel farther horizontally, but they will not go as high for the maximum height.

Time to reflect and share….

- Each pair of students will present their findings to the class. The presentation should include, but is not limited to, what you discovered during this activity, a discussion of how your group determined the horizontal distance and the maximum height for each of the three launches using the Geogebra files, any challenges that you encountered, and how this activity relates to the real-world besides launching rockets. Presentations should not exceed six minutes.

  The student activity sheet is very detailed in order to provide you with an example of what students might state in their presentation and in their summary. All calculations are shown for the same reason. Students may not include as much detail on the activity sheet, however, he/she should provide as much or more information and calculations in the presentation and summary.

- Each student should turn in a typed reflection summarizing your findings, all calculations, and what those calculations represent. Please understand that the summary is separate from the presentation, and your thoughts should be conveyed in your summary as well as in your presentation. In other words, they are each "stand alone pieces" of the project. Your summary should include what you discovered during this activity, a discussion of how you determined the horizontal distance and the maximum height for each of the three launches using the Geogebra files, if you agreed or disagreed with your original conjecture and why, a discussion of any challenges you encountered, how this activity relates to the real-world besides launching rockets, and a summary of what you learned during this project. The document should be typed in Microsoft Word, or similar program, using 12 point font, double spaced and should be a **minimum** of 300 words.

Students may mention the fields of engineering, physics, mathematics, and the military for how this activity relates to the real-world. Students may realize that the design, construction, and launch of the rocket are all important features that are explored in engineering and physics, but the angle of the launch and the effect it has on the horizontal distance travelled and the maximum height are very important details explored in the military.

Students may mention that finding the quadratic function using the Geogebra files was challenging; therefore, the teacher may have to show students that they can manually enter values by double-clicking on the "a", "h", or "k" and then type in the value. Also, students may find it challenging to approximate the height of the rocket after he/she launches it, in this case the teacher may want to encourage their partner to help them approximate the height by using a landmark in the classroom as a comparison.

***All files located on the following website: https://sites.google.com/site/rocketquadspba/home ***